# CS-472: Design Technologies for Integrated Systems

Exercise Problem Set 10 Solution — Date: 28/11/2024

Topic: Boolean methods (cf. slide set 12)

## Problem 1

Consider the logic network where inputs are $\{a, b, c, d\}$ and output is $\{f\}$ defined as:

$$\begin{aligned} k &= ad \\ n &= c + k \\ m &= \overline{(ab)} \\ f &= m + n \end{aligned}$$

Assuming $CDC_{in} = ab$, compute $CDC_{out}$.

*cf:* Algorithm in textbook p.385

*Ans:*

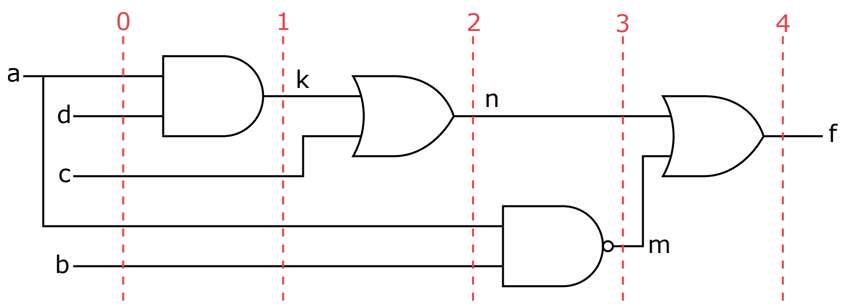


Figure 1: Cuts in the logic circuit during CDC computation

Figure **??** shows some possible succesive *cuts* of the network that we will use to compute the $CDC_{out}$ by network traversal. The $CDC$ computation for each cut proceeds as follows:

0. $CDC_{in} = ab$
1. $CDC_{cut} = ab + (ad \oplus k)$
   drop $d$: $(ab + (a \oplus k)) \cdot (ab + k) = ab + \bar{a}k$ $\quad$ ($\{a = 1, b = 1\}$ and $\{a = 0, k = 1\}$ will not happen)
2. $CDC_{cut} = ab + \bar{a}k + ((c + k) \oplus n)$
   drop $k$: $(ab + \bar{a} + \bar{n}) \cdot (ab + (c \oplus n)) = ab + c\bar{n} + \bar{a}\bar{c}n$
   drop $c$: $(ab + \bar{n}) \cdot (ab + \bar{a}n) = ab$

3. $CDC_{cut} = ab + (\overline{(ab)} \oplus m)$
   drop $a$: $(b + (\bar{b} \oplus m)) \cdot \bar{m} = \bar{m}$
   drop $b$: $\bar{m}$

4. $CDC_{cut} = \bar{m} + ((m + n) \oplus f)$
   drop $m$: $\bar{f}$
   drop $n$: $\bar{f}$
   $CDC_{cut} = \bar{f}$

Output $f$ is always $1$ since $m$ can never be $0$.

## Problem 2

Consider the logic network from Problem 1. Compute the ODC sets for all internal and input vertices assuming that the output is fully observable.

*cf:* Algorithm in textbook p.390
*Ans:*

$$
\begin{aligned}
\frac{\partial(m+n)}{\partial m} &= n \oplus 1 = \bar{n}, & ODC_m &= n = ad + c \\
\frac{\partial(m+n)}{\partial n} &= m \oplus 1 = \bar{m}, & ODC_n &= m = \overline{ab} \\
\frac{\partial(c+k)}{\partial k} &= c \oplus 1 = \bar{c}, & ODC_k &= c + m = c + \overline{ab} \\
\frac{\partial(c+k)}{\partial c} &= k \oplus 1 = \bar{k}, & ODC_c &= k + m = ad + \overline{ab} \\
\frac{\partial(ad)}{\partial d} &= 0 \oplus a = a, & ODC_d &= \bar{a} + c + m = \bar{a} + \bar{b} + c \\
\frac{\partial(\overline{ab})}{\partial b} &= 1 \oplus \bar{a} = \bar{a}, & ODC_b &= \bar{a} + n = \bar{a} + ad + c
\end{aligned}
$$

Signal $a$ has two fanouts: $\{k, m\}$ *cf:* p. 392

$$
\begin{aligned}
\frac{\partial(ad)}{\partial a} &= 0 \oplus d = d, & ODC_{a,k} &= \bar{d} + c + m = \bar{a} + \bar{b} + c + \bar{d} \\
\frac{\partial(\overline{ab})}{\partial a} &= 1 \oplus \bar{b} = \bar{b}, & ODC_{a,m} &= \bar{b} + n = ad + \bar{b} + c
\end{aligned}
$$

$$
\begin{aligned}
ODC_a &= ODC_{a,k}|_{a=\bar{a}} \,\overline{\oplus}\, ODC_{a,m} \\
&= (a + \bar{b} + c + \bar{d}) \,\overline{\oplus}\, (ad + \bar{b} + c) \\
&= (\bar{a}b\bar{c}d) \oplus (ad + \bar{b} + c) \\
&= \bar{b} + c + d
\end{aligned}
$$